

PALEY–WIENER TYPE THEOREMS ON THE HEISENBERG GROUP

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ABSTRACT. We review some results proved jointly with Francesca Astengo and Fulvio Ricci. Let $H_n \simeq \mathbb{C}^n \times \mathbb{R}$ be the $2n + 1$ real dimensional Heisenberg groups. The Fourier transform of an integrable function f on H_n is an operator valued function $\lambda \in \mathbb{R} \setminus \{0\} \mapsto \widehat{f}(\lambda)$. We are concerned with the behavior of $\widehat{f}(\lambda)$ when f is a Schwartz function or a smooth function with compact support on H_n . When f has some properties of invariance the expression of $f(\lambda)$ simplifies and it is possible to find nice characterizations of the Fourier transform. In particular if f is a $U(n)$ -invariant function, then \widehat{f} can be viewed as a scalar function defined on a subset Σ of \mathbb{R}^2 , the so called *Heisenberg fan*. We proved that the Fourier transform is an isomorphism from the space of $U(n)$ -invariant Schwartz functions onto the space of restrictions to Σ of Schwartz functions on \mathbb{R}^2 . In this talk we mainly describe extensions of this result and real Paley-Wiener theorems for smooth compactly supported functions.